List of topics for the preliminary exam in analysis

1 Topological concepts

1. Topological spaces.
2. Metric spaces, continuity and compactness, locally compact spaces.
3. Urysoh's Lemma, the Tietze Extension Theorem, Tychonov's Theorem, Stone-Weierstrass Theorem, Arzelà-Ascoli Theorem.

2 Measure theory

1. σ-algebras, the Borel σ-algebra, continuity properties of measures, complete measures. Measurable functions, properties of measurable functions, approximation properties of measurable functions, the Cantor-Lebesgue function.
2. Lebesgue measure, Lebesgue-Stieltjes measure.
3. Regularity properties of measures, Radon measures.
4. Existence of sets that are not Lebesgue measurable.
5. Signed measures, Hahn decomposition, Jordan decomposition, complex measures, total variation, the Lebesgue-Radon-Nikodym Theorem, applications.
6. Extension of measures, product measures.
7. The Riesz Representation Theorem for positive and complex measures.

3 Integration theory, convergence results

1. The Monotone Convergence Theorem, Fatou's Lemma, the Dominated Convergence Theorem, parameter integrals.
2. The Lebesgue integral versus the Riemann integral.
3. L^p spaces and duality.

4. Pointwise convergence, convergence almost everywhere, almost uniform convergence, convergence in measure, L^p-convergence, weak convergence.

5. Egorov's Theorem, Lusin's Theorem, and approximation properties of integrable functions.

6. The Fubini-Tonelli Theorem for product measures.

7. The Transformation Theorem for integrals and applications.

8. Integration on locally compact Hausdorff spaces. The Daniell integral.

4 Differentiation of functions and measures

1. The Lebesgue Differentiation Theorem, the Lebesgue set of a function, differentiation of measures.

2. Functions of bounded variation, Jordan decomposition, differentiation of functions of bounded variation.

3. Absolutely continuous functions, the Fundamental Theorem for Lebesgue integrals, convex functions.

5 Convolution and approximation

1. Convolution of functions, mapping properties for the convolution.

2. Mollifiers, approximate identities, smooth partitions of unity, density theorems in L^p, the Weierstrass Theorem.

6 Topics in Functional Analysis

1. Banach spaces: bounded linear operators, compactness and finite dimensional vector spaces.

2. The Baire Category Theorem, the Open Mapping Theorem, the Closed Graph Theorem.

3. The Hahn-Banach Theorem and consequences.

4. Hilbert spaces: best approximations, the representation theorem for bounded linear functionals, Bessel's inequality, orthonormal bases.